Frequency-Dependent Characteristics of Current Distributions on Microstrip Lines

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Abstract —**The spectral-domain analysis using Chebyshev's polynomials as basis functions is used to obtain the frequency-dependent characteristics of current distributions and the effective relative permittivities of an open microstrip line. The results obtained are compared with other available results. To accurately obtain the current distributions requires a larger number of basis functions. Both longitudinal and transverse current distributions on the strip are shown for wide ranges of frequency ($0 \leqslant h/\lambda_0 \leqslant 1$).**

I. Introduction

The dispersion characteristics of microstrip lines have been investigated by a number of researchers using a variety of methods (see [1]–[10] and references therein). These methods are intrinsically rigorous. However, the numerical results shown in many papers were calculated by expressing the current distributions with a small number of basis functions to save CPU time. The current distributions are fundamental quantities as sources for electromagnetic fields of microstrip lines. However, the literature on the determination of these distributions is sparse [11]–[14].

Recently, Shih *et al.* [13] proposed a full-wave analysis based on conformal mapping and variational reaction theory. For a number of cases, the results of effective relative permittivities [13] were in good agreement with those tabulated in [8]. Furthermore, Shih *et al.* [13] revealed the frequency dependence of the current distributions. This was the first time that those characteristics were reasonably obtained for wide ranges of frequency. At lower frequency, the results agreed well with those shown in [11]. Subsequently, Faché and De Zutter [14] showed the frequency-dependent characteristics of the current distributions. However, in both these papers these characteristics were not shown in frequency ranges higher than $h/\lambda_0 = 0.2$.

The present article shows the frequency-dependent characteristics of current distributions and the effective relative permittivities up to $h/\lambda_0 = 1$ for several cases. These results are obtained by the spectral-domain approach [4] using the Chebyshev polynomials adopted by Kitazawa and Hayashi [6] as basis functions. The results obtained are compared with other available results.

II. Basis Functions

The open microstrip line under consideration is shown in Fig. 1. It is assumed to be uniform and infinite in both the x and the z direction. The infinitesimally thin strip and the ground plane are taken as perfect conductors. The structure is divided into two regions, corresponding to the air and the dielectric structure. It is also assumed that the substrate material is lossless and that its relative permittivity and permeability are ϵ and μ ($=1$), respectively.

The propagation constant β can be obtained by the spectral-domain approach [4] following the corrections mentioned in [8]. Then the choice of the basis functions is important for numerical efficiency. Kitazawa and Hayashi [6] adopted $T_n(2x/w)/\sqrt{1-(2x/w)^2}$ and $U_n(2x/w)$ as basis functions and analyzed various types of striplines by the network analytical method of electromagnetic fields; $T_n(x)$ and $U_n(x)$ are Chebyshev polynomials of the first and second kinds, respectively. Kitazawa and Hayashi claimed that fast convergence to the exact values for $\epsilon_{\mathrm{eff}}(f)$ was obtained, even for the cases of $M = N = 2$, up to a normalized frequency of $h/\lambda_0 = 0.16$ [6]. They showed the results of $\epsilon_{\mathrm{eff}}(f)$ but not for the current distributions.

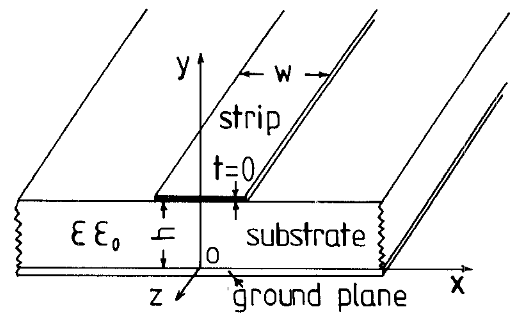

Fig. 1. Microstrip configuration.

TABLE I
Comparison of Effective Relative Permittivities ($\epsilon = 8$, $w/h = 1$)

METHOD \ h/λ_0		0.005	0.05	0.1	0.2	0.3	0.4	0.7	1 0
[8]		5.468	6.124	6.742	7.361	7.620	7.747	7.898	7 945
KI	M(N)								
	1	5.4662	6.1218	6.7462	7.3693	7.6302	7.7561	7.8993	7.9458
	2	5 4678	6.1272	6.7576	7.3996	7.6591	7.7826	7.9164	7 9562
	3	5.4678	6 1272	6.7576	7.3996	7.6591	7.7826	7.9164	7.9562
	4	5.4678	6.1272	6.7576	7.3996	7.6591	7.7826	7.9164	7.9562
[13]		5 471	6 130	6.753	7.393	7 654	7.778	7.914	7 948

KI: Present method; $h/\lambda_0 = h(\mathrm{mm})f(\mathrm{GHz})/299.7925$.

For the dominant mode, it is easily seen that $I_z(x)$ is even-symmetric with respect to the $y-z$ plane while $I_x(x)$ is odd-symmetric [4]. $T_n(x)(U_n(x))$ is the even (odd) function when n is an even number and the odd (even) function when n is an odd number. Therefore, the present article takes the following basis functions for $I_{xn}(x)$ and $I_{zn}(x)$:

$$I_{xn}(x) = U_{2n}(2x/w) \tag{1a}$$

$$I_{zn}(x) = T_{2(n-1)}(2x/w)\Big/\sqrt{1-(2x/w)^2} \tag{1b}$$

on the strip ($|x| \leqslant w/2$) and zero for $|x| > w/2$. These basis functions are Fourier transformed as follows:

$$\tilde{I}_{xn}(\alpha) = j(-1)^{(n-1)/2}\frac{n\pi}{\alpha}J_n\left(\frac{|\alpha|w}{2}\right) \tag{2a}$$

$$\tilde{I}_{zn}(\alpha) = (-1)^{(n-1)/2}\frac{\pi w}{2}J_{n-1}\left(\frac{|\alpha|w}{2}\right) \tag{2b}$$

where $J_n(x)$ is the nth-order Bessel function.

III. Numerical Results

Table I shows a comparison of effective relative permittivities $\epsilon_{\mathrm{eff}}(f)$ ($\epsilon = 8$, $w/h = 1$) for Kobayashi *et al.* [8], the present method, and Shih *et al.* [13]. The influence of the number of basis functions M ($=N$) on the current components is also included in Table I. M and N denote the numbers of basis functions for

Manuscript received July 21, 1988; revised November 4, 1988.
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IEEE Log Number 8826043.

TABLE II
COMPARISON OF EFFECTIVE RELATIVE PERMITTIVITIES ($\epsilon = 8$, $w/h = 0.1$)

h/λ_0 \ METHOD	0	0 005	0 05	0.1	0.2	0 3	0 4	0 7	1.0
[8]	4.92578*	4.937	5 270	5 765	6.799	7.367	7 633	7 876	7.939
KI	4 92578*	4.937	5 270	5.765	6.799	7 376	7 633	7 876	7.939
[13]	5.02111	5.033	5 384	5.863	6.791	7 365	7.617	7.872	7.938

KI: Present method; *: [15]

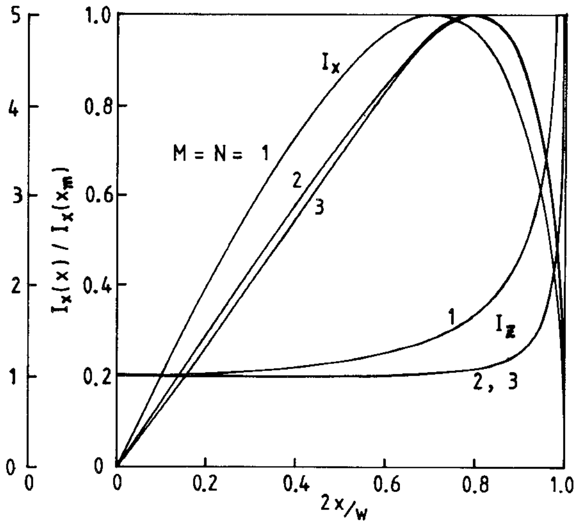

Fig. 2 Normalized current distributions versus normalized distance with the numbers of basis fucntions as parameters ($\epsilon = 8$, $w/h = 1$, $h/\lambda_0 = 0.2$)

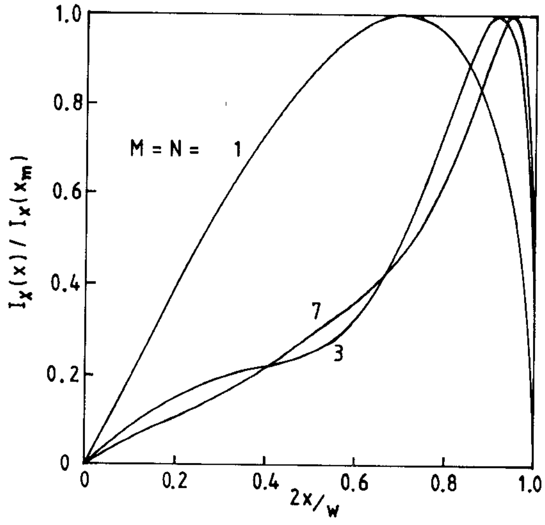

Fig. 3. Normalized transverse current distributions versus normalized distance with the numbers of basis functions as parameters ($\epsilon = 8$, $w/h = 1$, $h/\lambda_0 = 1$).

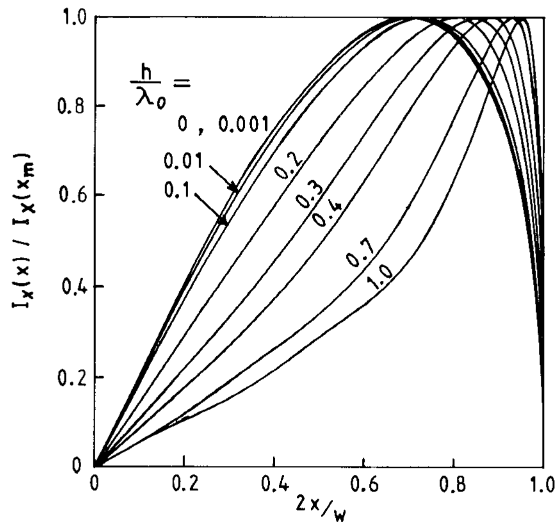

Fig 4 Frequency-dependent characteristics of normalized transverse current distributions ($\epsilon = 8$, $w/h = 1$)

the transverse and longitudinal current components, respectively. Table I shows that fast convergence to the exact values of $\epsilon_{eff}(f)$ is obtained even for the cases of $M = N = 2$. The results obtained with the present method show that the results in [8] and in [13] are accurate to better than 0.5 and 0.1 percent, respectively. The results in [13] were in good agreement with those in [8] for many cases. However, there were discrepancies up to about 2 percent at lower frequencies for the narrower case of $\epsilon = 8$ and $w/h = 0.1$. This discrepancy of 2 percent was seen even at $h/\lambda_0 = 0$. This comparison is reproduced here in Table II. The results with the present method are obtained for $M = N = 2$. The result $\epsilon_{eff}(0) = 4.92578$ marked by an asterisk in Table II was calculated by the Green's function technique with an extremely high degree of accuracy [15]. The results with the present method are in very good agreement with those in [8]. These facts show that the calculation using the method given by Shih *et al.* [13] must be carried out carefully for the cases involving narrower strips, for example, $w/h = 0.1$.

Fig. 2 shows the normalized longitudinal and transverse current distributions for several values of the number of basis functions ($M = N = 1,2,3$) at the normalized frequency $h/\lambda_0 = 0.2$ for the case of $\epsilon = 8$ and $w/h = 1$. A good convergence of current distributions requires $M = N = 3$ for a normalized transverse current distribution $I_x(x)/I_x(x_m)$ and $M = N = 2$ for a normalized longitudinal current distribution $I_z(x)/I_z(0)$. $I_x(x_m)$ denotes the extremum value of $I_x(x)$, and $I_z(0)$ the value of $I_z(x)$ at $x = 0$. It is confirmed, although not shown here, that the above requirements with respect to the numbers M and N are also valid for the cases where h/λ_0 is less than 0.2. However, larger values of M and N are required for the convergence of a current distribution for cases where h/λ_0 is higher than 0.2.

To illustrate this, the normalized transverse current distributions at $h/\lambda_0 = 1$ for the case of $\epsilon = 8$ and $w/h = 1$ are shown for $M = N = 1,3,7$ in Fig. 3. It is seen in this figure that $M = N = 3$ is insufficient for the case where $h/\lambda_0 = 1$. The curve for $M = N = 5$ is in agreement with that for $M = N = 7$ although it is not shown in Fig. 3. The present article takes $M = N = 3$ for $h/\lambda_0 \leq 0.2$, $M = N = 4$ for $0.2 < h/\lambda_0 \leq 0.4$, and $M = N = 5$ to 7 for $h/\lambda_0 > 0.4$ to accurately obtain the current distributions.

Fig. 4 shows the frequency-dependent characteristics of the normalized transverse current distributions for the case where $\epsilon = 8$ and $w/h = 1$. The curve for $h/\lambda_0 = 0$ obtained in [11] cannot be distinguished from that for $h/\lambda_0 = 0.001$. It is seen in Fig. 4 that the point x_m giving the extremum of the transverse current distribution shifts toward the strip edge.

Fig. 5 shows the frequency-dependent characteristics of the normalized longitudinal current distributions for the case where $\epsilon = 8$ and $w/h = 1$. The curve for $h/\lambda_0 = 0$ obtained in [11] is given by the dashed lines in Fig. 5 and is the upper bound for the curves of nonzero frequencies. The distribution curve for h/λ_0 higher than about 0.2 begins to have the part below the horizontal line of $I_z(x)/I_z(0) = 1$.

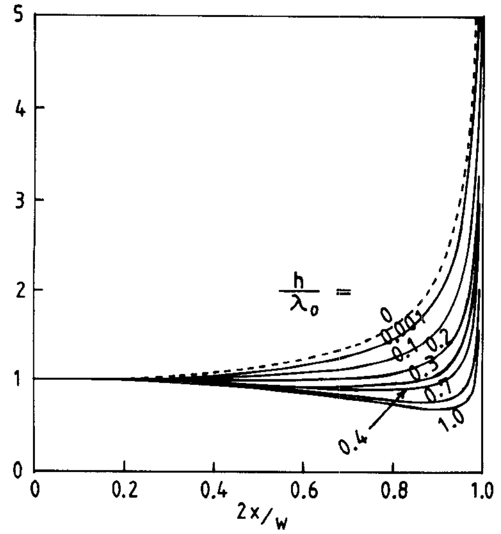

Fig. 5. Frequency-dependent characteristics of normalized longitudinal current distributions ($\epsilon = 8$, $w/h = 1$). --- Green's function technique [11]; — present method.

The shifts of the current distributions with respect to frequencies for $h/\lambda_0 \leqslant 0.2$ shown in Figs. 4 and 5 are similar to those revealed by Shih *et al.* [13], although for cases where ϵ and w/h have values different from those of the present article.

IV. Conclusion

The spectral-domain approach has been used to obtain the frequency-dependent characteristics of current distributions and the effective permittivities of open microstrip lines. The functions $U_{2n}(2x/w)$ and $T_{2(n-1)}(2x/w)/\sqrt{1-(2x/w)^2}$ have been adopted as basis functions; $T_n(x)$ and $U_n(x)$ are Chebyshev polynomials of the first and second kinds, respectively. Numerical results reported in this article have been compared with other available data.

References

[1] E. J. Denlinger, "A frequency dependent solution for microstrip transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 30–39, Jan. 1971.
[2] Y. Fujiki, Y. Hayashi, and M Suzuki, "Analysis of strip transmission lines by iteration method," *J. Inst. Electron. Commun. Eng. Japan*, vol. 55-B, pp. 212–219, May 1972 (in Japanese); *Electron Commun Japan*, vol. 55, pp. 74–80, May 1972.
[3] G. Kowalski and R Pregla, "Dispersion characteristics of single and coupled microstrips," *Arch. Elek, Übertragung.*, vol 26, pp. 276–280, June 1972.
[4] T. Itoh and R. Mittra, "Spectral-domain approach for calculating the dispersion characteristics of microstrip lines," *IEEE Trans Microwave Theory Tech.*, vol. MTT-21, pp. 496–499, July 1973.
[5] E. F. Kuester and D. C. Chang, "An appraisal of methods for computation of the dispersion characteristics of open microstrip," *IEEE Trans Microwave Theory Tech.*, vol. MTT-27, pp. 691–694, July 1979.
[6] T. Kitazawa and Y. Hayashi, "Propagation characteristics of striplines with multilayered anisotropic media," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 429–433, June 1983.
[7] N. G. Alexopoulos, "Integrated-circuit structures on anisotropic substrates," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 847–881, Oct. 1985
[8] M. Kobayashi and F. Ando, "Dispersion characteristics of open microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 101–105, Feb 1987.
[9] B. M. Sherril and N. G. Alexopoulos, "The method of lines applied to a finline/strip configuration on an anisotropic substrates," *IEEE Trans Microwave Theory Tech*, vol. MTT-35, pp. 568–574, June 1987
[10] B. E. Kretch and R. E. Collin, "Microstrip dispersion including anisotropic substrates," *IEEE Trans. Microwave Theory Tech*, vol MTT-35, pp. 710–718, Aug. 1987.
[11] M. Kobayashi, "Longitudinal and transverse current distributions on microstriplines and their closed-form expression," *IEEE Trans Microwave Theory Tech*, vol. MTT-33, pp 784–788, Sept. 1985
[12] M. Kobayashi and H. Momoi, "Longitudinal and transverse current distributions on coupled micristrip lines," *IEEE Trans. Microwave Theory Tech.*, vol 36, pp. 588–593, Mar. 1988.
[13] C. Shih, R. B. Wu, S. K. Jeng, and C. H. Chen, "A full-wave analysis of microstrip lines by variational conformal mapping technique," *IEEE Trans Microwave Theory Tech*, vol. 36, pp. 576–581, Mar. 1988.
[14] N. Faché and D. D Zutter, "Rigorous full-wave space-domain solution for dispersive microstrip lines," *IEEE Trans Microwave Theory Tech.*, vol. 36, pp. 731–737, Apr. 1988
[15] M. Kobayashi, "Analysis of the microstrip and the electrooptic light modulator," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 119–126, Feb. 1978.

Shift of the Complex Resonance Frequency of a Dielectric-Loaded Cavity Produced by Small Sample Insertion Holes

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Abstract —**The presence of small sample insertion holes in a cylindrical cavity produces a shift in the complex resonance frequency of the cavity. A mathematical model is proposed to compute the shift when the cavity oscillates in an axially symmetric TM_{0mp} mode. The treatment applies to samples with arbitrary complex permittivity. The model is compared with other treatments and checked against measured results.**

I. Introduction

Insertion holes in resonant cavities produce changes in both the real and imaginary parts of the complex resonance frequency, which may amount to a few percent and are significant in high-precision measurements. Several attempts have been made to quantify hole effects. Estin and Bussey [1] and Meyer [2] have estimated the change in the real part of the resonance frequency for some simple TM_{0mp} modes. Their main assumptions were that the field is not perturbed in the main body of the cavity and that in the tubes it is well represented by the first evanescent TM mode. More recently, Li and Bosisio [3] have significantly improved the treatment by allowing for a large number of modes in the tubes. They have obtained correction terms due to insertion holes for both the real part of the resonance frequency and the quality factor of the cavity.

The present paper is an attempt to compute the shift of the complex resonance frequency of a cavity produced by small sample insertion holes. It was largely inspired by the work of Li and Bosisio, which it tries to improve in two different ways. First, we take fully into account the fact that, for lossy samples, the phasors and the wavenumbers in the tubes are genuinely complex. Second, we carry a larger fraction of the calculations analytically. The resulting formulas are less susceptible to numerical errors.

Manuscript received November 6, 1987; revised October 24, 1988.
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IEEE Log Number 8826051.